

Locality hypothesis and the speed of light

G.Longhi

Physics Department, University of Florence

(Dated: 12 December 2005)

The locality hypothesis is generally considered necessary for the study of the kinematics of non-inertial systems in Special Relativity. In this paper we discuss this hypothesis, showing the necessity of an improvement, in order to get a more clear understanding of the various concepts involved, like coordinate velocity and standard velocity of light. Concrete examples are shown, where these concepts are discussed.

PACS numbers: 03.30.+p

I. INTRODUCTION

The study of the relativistic kinematics in non-inertial reference frames has received recently much attention [1], [2], [19]. Almost always the study of these frames is based on the so called "locality hypothesis" which is the following statement:

a) "an accelerated observer is equivalent to an infinite sequence of hypothetical inertial observers along its world-line, each momentarily co-moving with the accelerated observer," [4]. We can add the most obvious specification that the observer should be a "point-like observer".

Another formulation of this hypothesis is given by:

b) "locally, neither gravity nor acceleration changes the length of a standard rod or the rate of a standard clock relative to a nearby freely falling standard rod or a standard clock instantaneously co-moving with it". And still "Stated another way, a local inertial observer is equivalent to a local co-moving non-inertial observer in all matters having to do with measurements of distance and time" [5].

Let us quote even another formulation

c) "the speed of light, as measured *locally* by means of standard rods and clocks at rest with respect to the non-inertial observer should be exactly the same as that observed in the local inertial frame, the latter being *c* in both directions" [6].

Even if these formulations are all correct, some ambiguity can arise if we do not clearly separate the concepts of standard clocks and rods from that of coordinate times and lengths. This because these last quantities are in general quite different from the standard ones.

For instance, in reference [5], it is claimed that the postulate, which says that the speed of light is the same for all inertial observers and equal to *c* (relativity postulate), could be violated in rotating frames, if we maintain the locality hypothesis; but again we observe that no clear distinction is made there between the coordinates of the accelerated system and that of the local inertial system.

Indeed, as will be seen in Section VI, if we maintain this distinction, no contradiction arises and no violation of the relativity postulate or of the locality hypothesis is required.

Now, in order to have a clear understanding of the problem, it seems necessary to give in this context a more precise notion of the aforementioned equivalence or, which is the same, a clear relation between the coordinates of a generally non-inertial system and the local inertial coordinates.

In this work we want to recover a precise definition of this equivalence and to show how it works in some examples.

In Section II we will recall a well known fact about the mathematics of Riemann geometry, namely that is always possible to transform a given metric tensor to a new one, which is locally Minkowskian and with zero first derivatives. In this way it is possible to define a local inertial frame, which can be used to define the proper time and the proper lengths.

Of course this definition is well known, see for instance the reference [16] or [7].

With these results we can discuss the possibility of an anisotropic propagation of light, and its relation with the propagation as seen from the co-moving local inertial frame.

In Section III we define some notation. In Section IV we study the example of a Galilean transformation. This is the most simple example, but it has already interesting characteristics regarding the propagation of light.

In Section V we study the hyperbolic motion, and in Section VI the rotating disk.

In all these examples we look only at the propagation of light and not of particles, this because the study of the propagation of a material body would require the complete solution of the geodesic equations. This could be done, but it seems unnecessary for a clear understanding of this matter.

In the examples that will be discussed, the existence of a reference frame, inertial and with Minkowskian coordinates, called laboratory frame, will be assumed. In this way the accelerated systems will be easily defined, but it must be understood that its existence is not strictly necessary for the definition of the local inertial frame.

In Section VII is devoted to an application: the solution of the so called Selleri's paradox.

We end in Section VIII with a concluding remark.

II. THE LOCALITY HYPOTHESIS

Let us start with a well known fact about Riemannian geometry: given a metric tensor g in a given reference frame with coordinates $\{x^\mu\}$, free of singularities and not degenerate, it is always possible, in the neighborhood of a given point P_o , to find a transformation to a new set of coordinates $\{x'^\mu\}$, such that g will transform in a Minkowski metric in the point P_o , and such that its first derivatives in this point be zero (normal coordinates). In general it is not possible to require the vanishing of the higher derivatives of g .

This transformation is determined up to a Lorentz transformation [7], [16].

Following [16], but see also [7] and [9], this transformation can be written

$$x'^\mu = b^\mu_\alpha (x^\alpha - x^\alpha_o) + \frac{1}{2} b^\mu_\nu \Gamma^\nu_{\alpha\beta}(x^\alpha - x^\alpha_o)(x^\beta - x^\beta_o), \quad (1)$$

where $\{x^\mu_o\}$ are the coordinates of P_o , which is the origin of the new coordinates $\{x'^\mu\}$, and $\Gamma_o \equiv \Gamma(P_o)$ are the connection coefficients of the metric tensor g at the point P_o . We will always use coordinate basis for the vector fields, so that the connection coefficients Γ are given by

$$\Gamma^\mu_{\alpha\beta} = \frac{1}{2} g^{\mu\nu} \left(\frac{\partial g_{\nu\alpha}}{\partial x^\beta} + \frac{\partial g_{\nu\beta}}{\partial x^\alpha} - \frac{\partial g_{\alpha\beta}}{\partial x^\nu} \right). \quad (2)$$

The matrix b must be chosen such that

$$\eta_{\alpha\beta} b^\alpha_\mu b^\beta_\nu = g_{\mu\nu}(P_o), \quad (3)$$

where η is the metric of Minkowski, with signature $(-, +, +, +)$.

It can be shown that the matrix transformation (3) is always possible [8], and we will see it explicitly in the examples of the following Sections.

Once g is reduced to η in P_o we have clearly still the freedom to perform an arbitrary Lorentz transformation. But this can be uniquely determined if we require in addition that the coordinates axis be tangent to a given tetrad of vectors in P_o . In particular the coordinate axis of the time $x'^0 = ct'$ will be chosen tangent to the axis of $x^0 = ct$, in this way the frame defined by the new coordinates will be a system co-moving with P_o .

In this sense we may say that the transformation $\{x^\mu\} \rightarrow \{x'^\mu\}$ is unique.

Let us now look at the geodesic equations. These are

$$\frac{d^2 x^\mu}{d\lambda^2} + \Gamma^\mu_{\alpha\beta} \frac{dx^\alpha}{d\lambda} \frac{dx^\beta}{d\lambda} = 0, \quad (4)$$

and, in the case of null geodesics, we must add

$$g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda} = 0, \quad (5)$$

with λ some parameter.

These are the geodesic equations in the $\{x^\mu\}$ coordinates. But in terms of the $\{x'^\mu\}$ coordinates in the point P_o these equations become

$$\begin{cases} \frac{d^2 x'^\mu}{d\lambda^2} |_{P_o} = 0, \\ \eta_{\mu\nu} \frac{dx'^\mu}{d\lambda} \frac{dx'^\nu}{d\lambda} |_{P_o} = 0, \end{cases} \quad (6)$$

since in P_o the connection coefficients Γ_o , being proportional to the first derivatives of the metric tensor, are zero.

So the geodesic equations in the new coordinates give rise to a light propagation isotropic and along trajectories, which are rectilinear in a second order neighborhood of the point P_o .

More exactly, the world line with λ as a parameter will be of the form

$$x'^\mu = a^\mu + b^\mu \lambda + O(\lambda^3), \quad (7)$$

since the second order term is zero due to equation (6). This equation determines the degree of approximation of the local inertial frame to the system S.

On the other hand, the new metric tensor $g'_{\mu\nu} dx'^\mu dx'^\nu$ will be inertial only up to first order in $x^\mu - x^\mu_o$, as seen from (1).

If we now apply the previous result to the relativistic kinematics of a non-inertial frame with a generic metric tensor g , we see that it is quite natural to define the aforementioned equivalence in terms of the transformation $\{x^\mu\} \rightarrow \{x'^\mu\}$. So we can change the locality hypothesis (a) to the following form:

"an accelerated observer is equivalent, by means of a sequence of coordinate transformation (1), to an infinite sequence of inertial observers along its world-line, each momentarily co-moving with the accelerated observer."

If we agree that proper times and distances are the times and distances measured by standard clocks and rods, that is clocks and rods at rest in a given place and at a given time on the accelerated frame, we can interpret this hypothesis by saying: the measure of proper times and distances in a given accelerated system are given by the times and the distances as measured by using the coordinates of the local inertial frame, which is co-moving with the accelerated system, at the same place and time.

We will see that ambiguities in the application of the principles of relativity, in situations in which the propagation of light is anisotropic in one reference frame and isotropic in another, will be in principle solved.

This will be essentially achieved by observing that two observers, practically in the same place and at the same time, one performing measures of time and distances in terms of the $\{x^\mu\}$ coordinates and the other in terms of the $\{x'^\mu\}$, will see a different propagation of light.

We will see in detail this fact in the examples of the following Sections, but we may easily understand how it

may happen by considering a 2-dimensional system described by a set of coordinates $\{cT, X\}$ with Minkowskian metric

$$\eta = \eta_{\mu\nu} dX^\mu dX^\nu = -c^2 dT^2 + dX^2. \quad (8)$$

The light-cone equation with the vertex in the origin is, of course

$$-c^2 T^2 + X^2 = 0. \quad (9)$$

If we perform a linear transformation to a new set of coordinates $\{cT, X\} \rightarrow \{ct, x\}$, which is not a Lorentz transformation, and if we choose this transformation such that to give a mixing of the spatial and the temporal coordinates, the new metric will be not time-orthogonal.

Of course, any transformation regular enough can be Taylor expanded, and can be considered a linear transformation in the neighborhood of a given point in some approximation.

The result of this transformation will be a deformation of the light cone in such a way that the speed of light in the forward direction is, say, higher than c and in the backward direction less than c . If for instance this linear transformation is (Galilean transformation)

$$\begin{cases} t &= T, \\ x &= X - vT, \end{cases} \quad (10)$$

the light-cone equation becomes

$$-c^2(1 - \beta^2)t^2 + 2vxt + x^2 = 0, \quad (11)$$

where $\beta = v/c$. The light-cone which was symmetric with respect to the time axis, is no more symmetric, and the speed of light is $c+v$ in the positive direction of the x axis and $-c+v$ in the other.

This situation is analogous to the propagation of the light from a star, coming from a visual position near to the sun. Suppose that this light is observed from a local inertial frame and from the astronomical non-inertial system. In the first case we have a rectilinear propagation with velocity c , in the second case we have a deflection [10].

In this example we have that the synchronization in the accelerated frame is determined by the laboratory frame, where the standard synchronization (Einstein synchronization) is supposed to hold. This will be true in all the examples considered in the following.

The composition of velocities that follows from this transformation is the Galilean composition law. Since the system is 1-dimensional there is no room for an explanation in terms of a normal velocity, as done in reference [11], on the contrary, this example shows that the explanation given in that reference cannot be maintained. As

any other transformation it has its own composition law. We will study this transformation in more details below.

Several topics, which could be studied using the transformation (1), will not be discussed. For instance, the spatial geometry, that is the surface of constant proper time, or the limit of validity of the locality hypothesis, when interpreted in the large, as done in [4], [25] will not be studied.

III. THE CO-MOVING LOCAL INERTIAL FRAME.

In the examples of the following Sections we will use the following notations: the coordinates of a inertial laboratory frame \mathcal{L} will be denoted $\{X^\mu\}$, with $X^0 = cT$. It is tacitly supposed that in \mathcal{L} the clocks are synchronized according to the Einstein rule [13]. The metric tensor will be

$$\eta_{\mu\nu} dX^\mu dX^\nu = -c^2 dT^2 + dX^2 + dY^2 + dZ^2, \quad (12)$$

and an accelerated system \mathcal{S} will be defined with respect to this system.

We must stress that the existence of the frame \mathcal{L} is not necessary, but it is useful for deducing the metric of \mathcal{S} .

The coordinates of the accelerated system \mathcal{S} will be denoted $\{x^\mu\}$, with $x^0 = ct$.

The synchronization of clocks in \mathcal{S} is not arbitrary, but it is borrowed from that of \mathcal{L} . Usually the synchronization is characterized by the quantity ϵ [14] [15], see the Appendix, which, in the case of the Einstein synchronization, is $1/2$. In \mathcal{S} ϵ can be different from $1/2$.

Finally, for the co-moving local inertial system \mathcal{L}' , we will use the coordinates $\{x'^\mu\}$, with origin in the generic point $P_o \equiv \{x_o^\mu\}$. This system will be determined by the transformation $\{x^\mu\} \rightarrow \{x'^\mu\}$ and it will be connected to the system \mathcal{L} by means of a inhomogeneous Lorentz transformation.

Now, the value of $\epsilon = 1/2$ is conserved under Lorentz transformation. This means that, if we define the $\{x'^\mu\}$ in terms of the $\{X^\mu\}$, since these coordinates are connected by a Lorentz transformation, the synchronization in \mathcal{L}' will be still with $\epsilon = 1/2$, and this will be true for each local inertial frame.

An observation which is almost obvious, is the following: if we want to consider the transformation from the system \mathcal{L} to the system \mathcal{S} as a physical transformation, and not merely as a change of notations, we must suppose that experimental procedures for the measure of the \mathcal{S} coordinates be provided. So it must be possible the measure of the speed $\frac{dx}{dt}$, which will be called coordinate speed [11],[12]. This seems obvious, but it can be practically difficult in some situation. For instance, in the case of a

rotating disk it can be not easy to measure the coordinate time t , by an observer rotating with the disk, since it is quite different from the proper time.

The \mathcal{S} coordinates will be called the coordinate time and space respectively, while the coordinates $\{x'^\mu\}$ of \mathcal{L}' are by definition the proper time (x'^0), that is the time measured with a standard clock in P_o , and the proper space (\vec{x}'), measured with a standard rod in the same place.

The speed $\frac{d\vec{x}'}{dt'}$ will be called the standard speed. For what we have said before the standard speed of light will be always isotropic and of value c , while the coordinate speed can be anisotropic and different from c . It is the standard speed that is measured locally by standard clocks and rods.

IV. THE GALILEAN TRANSFORMATION

Let us consider again the Galilean transformation from our laboratory frame \mathcal{L} to \mathcal{S} , defined as in [17]

$$\begin{cases} cT \rightarrow ct = cT, \\ X \rightarrow x = X - vT, \\ Y = y, \quad Z = z. \end{cases} \quad (13)$$

In this Section the coordinates Y, y and Z, z will be omitted.

The metric tensor in the new coordinates is

$$g = -c^2(1 - \beta^2)dt^2 + dx^2 + 2\beta dxcdt, \quad (14)$$

where $\beta = v/c$.

Since the metric components are constant, the connection coefficients Γ will be zero, as a consequence the geodesic equation in \mathcal{S} will be as

$$\frac{d^2x^\mu}{d\lambda^2} = 0. \quad (15)$$

Nevertheless, as we have seen, the propagation of light is anisotropic. Indeed, for the propagation of light we have, from (14),

$$-c^2(1 - \beta^2)dt^2 + dx^2 + 2\beta dxcdt = 0, \quad (16)$$

or

$$-c^2(1 - \beta^2) + \dot{x}^2 + 2\beta c\dot{x} = 0, \quad (17)$$

where $\dot{x} = \frac{dx}{dt}$, from which

$$\dot{x} = \pm c - v. \quad (18)$$

The synchronization parameter is $\epsilon = \frac{1}{2}(1 \pm \beta)$.

Let us consider an event P_o fixed in the \mathcal{S} system, with coordinates (t_o, x_o) . In the \mathcal{L} system it will describe a world-line

$$X = x_o + vt, \quad \text{and} \quad T = t. \quad (19)$$

The vector e_o , defined as

$$e_o = \gamma\left(\frac{\partial}{\partial X^o} + \beta\frac{\partial}{\partial X}\right), \quad \text{with} \quad e_o^2 = -1, \quad (20)$$

where $\gamma = (1 - \beta^2)^{-\frac{1}{2}}$, is tangent to the world line, and the vector e_1

$$e_1 = \gamma\left(\beta\frac{\partial}{\partial X^o} + \frac{\partial}{\partial X}\right), \quad \text{with} \quad e_1^2 = 1, \quad (e_1 | e_o) = 0, \quad (21)$$

where $(|)$ is the metric scalar product and $X^o = cT$, is the vector orthogonal to e_o .

Let us now determine the transformation $x^\mu \rightarrow x'^\mu$, using equations (1) and (3). Since the Γ coefficients are zero we need only to find the matrix $b^\mu{}_\nu$. This is easily found:

$$\|b^\mu{}_\nu\| = \begin{pmatrix} \gamma(1 - \beta^2) & -\gamma\beta \\ 0 & \gamma \end{pmatrix}. \quad (22)$$

With this matrix we get for the coordinates x'^μ (with $x'^o = ct'$, $x'^1 = x'$, $x^o = ct$, $x \equiv x^1$)

$$\begin{cases} x'^o &= \gamma[(1 - \beta^2)(x^o - x_o^o) - \beta(x - x_o)], \\ x' &= \gamma(x - x_o). \end{cases} \quad (23)$$

It is easily verified that the new metric tensor g' is, as expected,

$$g' = -c^2 dt'^2 + dx'^2, \quad (24)$$

and, using both transformations (13) and (23), we may verify that the coordinate axis are tangent to the vectors e_o and e_1 :

$$\begin{cases} \frac{\partial}{\partial x'^o} &= e_o, \\ \frac{\partial}{\partial x'} &= e_1. \end{cases} \quad (25)$$

With these two conditions satisfied the transformation (23) is unique. The transformation (23) defines a comoving, local inertial frame in the neighborhood of the point P_o .

Finally we may determine the relation between the velocities. If u is a velocity as measured in \mathcal{S} and u' is

the corresponding velocity in \mathcal{L}' , we may easily find their relation from equations (23)

$$u' = \frac{u}{(1 - \beta^2) - \beta \frac{u}{c}}. \quad (26)$$

The velocity u , in the case of the velocity of the light, is given by (18), so we get

$$u' = \frac{\pm c(1 \mp \beta)}{1 - \beta^2 \mp \beta + \beta^2} = \pm c, \quad (27)$$

so the anisotropy of the propagation of light disappears in the local inertial frame.

In general it is also possible to define an "extended" frame, whose coordinates, up to second order in $x^\mu - x_o^\mu$, be identical with the $\{x'^\mu\}$ of equation (1). In the present case they are linear, so no approximation is needed and the coordinates given by equation (23) are the coordinates of the extended frame too. In any case, this extended frame could approximate the accelerated one only in a neighborhood of the point of interest.

V. THE HYPERBOLIC MOTION.

Following the same steps of the previous example, we may study the hyperbolic motion in the plane (cT, X) [18].

Let us consider a congruence of time-like world lines given by

$$X = X_o + \frac{c^2}{g} (\sqrt{1 + (\frac{gT}{c})^2} - 1), \quad (28)$$

where g is a given acceleration and X_o is the initial value of X . Varying X_o we get the various world lines of the congruence. In the (T, X) plane, these lines are, as well known, hyperbolas.

In the limit $gT \ll c$ we get simply

$$X \simeq X_o + \frac{1}{2}gT^2. \quad (29)$$

We now consider a new set of coordinates obtained with the substitution $X_o \rightarrow x$ and with $T = t$:

$$\begin{cases} x &= X - \frac{c^2}{g} (\sqrt{1 + (\frac{gT}{c})^2} - 1), \\ t &= T. \end{cases} \quad (30)$$

The new set of coordinates (t, x) defines the accelerated system S . We could have expressed the time T in terms

of the proper time, as done in [26] and [27], but we prefer to maintain the Cartesian coordinates.

The initial Minkowskian metric η is transformed to

$$g = g_{\mu\nu} dx^\mu dx^\nu = -c^2(1 - \beta^2)dt^2 + dx^2 + 2c\beta dxdt, \quad (31)$$

where

$$\beta = \left(\frac{gt}{c}\right) \frac{1}{\sqrt{1 + (\frac{gt}{c})^2}}, \quad (32)$$

which is not time-orthogonal.

The vector tangent to these world-lines is

$$e_o = \gamma \left(\frac{\partial}{\partial X_o} + \beta \frac{\partial}{\partial X} \right), \quad (e_o)^2 = -1, \quad (33)$$

and the vector orthogonal to e_o is

$$e_1 = \gamma \left(\beta \frac{\partial}{\partial X_o} + \frac{\partial}{\partial X} \right), \quad (e_1)^2 = +1, \quad (e_1 | e_o) = 0. \quad (34)$$

In these equations

$$\gamma = \sqrt{1 + (\frac{gt}{c})^2} = (1 - \beta^2)^{-1/2}. \quad (35)$$

Even in this case we have an anisotropic propagation of light. Indeed, from equation (31) for the null geodesics

$$-c^2(1 - \beta^2) + \left(\frac{dx}{dt}\right)^2 + 2\beta c \frac{dx}{dt} = 0, \quad (36)$$

we get

$$\frac{dx}{dt} = \pm c(1 \mp \beta). \quad (37)$$

The synchronization parameter is $\epsilon = \frac{1}{2}(1 \pm \beta)$.

Now we want to determine the transformation $x^\mu \rightarrow x'^\mu$, in the neighborhood of a generic point $P_o \equiv (t_o, x_o)$. Since

$$\|g_{\mu\nu}\| = \begin{pmatrix} -(1 - \beta^2) & \beta \\ \beta & 1 \end{pmatrix}, \quad (38)$$

from (2) we get that the only connection coefficient different from zero is

$$\Gamma_{00}^1 = \frac{g}{c^2} (1 - \beta^2)^{3/2}. \quad (39)$$

The matrix $\|b_\nu^\mu\|$ is easily calculated

$$\|b^\mu{}_\nu\| = \begin{pmatrix} \gamma_o(1 - \beta_o^2) & -\gamma_o\beta_o \\ 0 & \gamma_o \end{pmatrix}, \quad (40)$$

where γ_\circ and β_\circ are γ and β in the point P_\circ .

Substituting this matrix and Γ in equation (1), we finally get (see also [27] where a transformation of the same kind is determined)

$$\begin{cases} x'^\circ &= \gamma_\circ \{ (1 - \beta_\circ^2) \eta^\circ - \beta_\circ \eta^1 - \frac{g\beta_\circ}{2c^2\gamma_\circ^3} (\eta^\circ)^2 \}, \\ x' &= \gamma_\circ \{ \eta^1 + \frac{g}{2c^2\gamma_\circ^3} (\eta^\circ)^2 \}, \end{cases} \quad (41)$$

where $x' \equiv x'^1$, $x^\circ = ct$ and $\eta^\mu = x^\mu - x_\circ^\mu$.

This is the transformation we were looking for.

We could as well have found the transformation $X^\mu \rightarrow x'^\mu$, using the relation (30). In this way it is possible to verify, with some work, that the coordinate axis of the new variables are tangent to the vectors e_\circ and e_1 in P_\circ :

$$\begin{cases} \frac{\partial}{\partial x'^\circ} &= e_\circ, \\ \frac{\partial}{\partial x'} &= e_1. \end{cases} \quad (42)$$

We may too verify that the new metric tensor g' is Minkowskian

$$g'_{\mu\nu}(x') dx'^\mu dx'^\nu = -c^2 dt'^2 + dx'^2, \quad (43)$$

which is true up to terms of order $O((x')^2)$, or even of order $O((\eta)^2)$, since the relation between the x' and the η is homogeneous.

More precisely we can write

$$\begin{cases} g'_{\mu\nu}(x') dx'^\mu dx'^\nu = [\eta_{\mu\nu} + O((x')^2)] dx'^\mu dx'^\nu = \\ = [g_{\mu\nu}(x) + O((\eta^\mu)^2)] dx^\mu dx^\nu. \end{cases} \quad (44)$$

where g is the metric (31). This means that the first order derivatives in P_\circ of g' are zero, as expected.

With the two conditions (42) and (43) the transformation (41) is unique.

From (41) we may get the relation between the velocities as seen from the two frames in the point P_\circ

$$\frac{dx'}{dt'} = \frac{\frac{dx}{dt}}{1 - \frac{\beta_\circ}{c} \frac{dx}{dt} - \beta_\circ^2} \quad (45)$$

We have seen before in equation (37) that the coordinate velocity $\frac{dx}{dt}$, measured in \mathcal{S} , is anisotropic. Substituting in the previous equation we obtain for the standard velocity $\frac{dx'}{dt'}$, measured in \mathcal{L}' in the point P_\circ

$$\frac{dx'}{dt'} = \pm c, \quad (46)$$

and there is no more anisotropy.

VI. THE ROTATING PLATFORM

The rotating platform is an example so much studied that we could hardly do justice to all contributors. We may refer for an extended set of references in the book edited by G.Rizzi and M.T.Ruggiero [19], where a consistent amount of bibliography can be found.

In any case we can quote here a set of articles on this subject, which have a point of view similar to that adopted here.

In [25] an analysis similar to our, with the end of verifying the limit of validity of the locality hypothesis, is performed for the rotating disk.

In [20] the synchronization of clocks on a rotating disk is discussed. On the other hand a discussion of all the aspect of the kinematics of the rotating disk can be found in [21], [22] and [23].

We do not discuss here the problem of the desynchronization of clocks, or Sagnac effect, for a round trip along a circumference of the disk. This because we directed our attention on the formulation of the locality hypothesis. This points are discussed in a lot of papers. Quoting some of them we have for instance [24], [29], [6], [28] and [5].

In order to avoid complications with the rigidity constraint, we will not study really a rotating disk, but rather we will consider a model defined by a congruence of time-like helices, given in the reference frame \mathcal{L} by the equations

$$\begin{cases} T &= t, \\ X &= x \cos(\omega t) - y \sin(\omega t), \\ Y &= x \sin(\omega t) + y \cos(\omega t), \end{cases} \quad (47)$$

where T, X, Y are the cartesian coordinates in \mathcal{L} , and x, y are the values of X and Y when the parameter t is zero. Nevertheless, for sake of simplicity and intuition, we will continue to speak of a disk, of the rim of this disk and so on.

The helices of the congruence are obtained by varying the values of x and y .

From now on we will omit the z coordinate.

The vector e_\circ tangent to the helix is given by

$$e_\circ \propto \dot{X}^\mu \frac{\partial}{\partial X^\mu}. \quad (48)$$

If we normalize it to -1 and use polar coordinates

$$X = R \cos(\Phi), \quad Y = R \sin(\Phi), \quad (49)$$

we get

$$\begin{cases} e_0 &= \gamma \left(\frac{\partial}{\partial X^\circ} + \frac{\omega}{c} \frac{\partial}{\partial \Phi} \right), \\ e_0^2 &= -1, \end{cases} \quad (50)$$

where $\gamma = 1/\sqrt{1-\beta^2}$, and $\beta = \omega r/c$.

We have two other vectors orthogonal each other and to e_0 ,

$$\begin{cases} e_1 &= \frac{\partial}{\partial R}, \\ e_2 &= \gamma(\beta \frac{\partial}{\partial X^\circ} + \frac{1}{R} \frac{\partial}{\partial \Phi}), \end{cases} \quad (51)$$

with

$$\begin{cases} e_1^2 = +1, & e_2^2 = +1, & (e_1|e_2) = 0, \\ (e_1|e_0) = (e_2|e_0) = 0. \end{cases}$$

The transformation to \mathcal{S} is given by

$$r = R, \quad \phi = \Phi - \omega T, \quad t = T, \quad (52)$$

where

$$x = r \cos(\phi), \quad y = r \sin(\phi), \quad (53)$$

In cartesian coordinates the transformation $(X, Y, T) \rightarrow (x, y, t)$ is

$$\begin{cases} x = X \cos(\omega T) + Y \sin(\omega T), \\ y = -X \sin(\omega T) + Y \cos(\omega T), \end{cases} \quad (54)$$

and $t = T$.

The metric tensor in the frame \mathcal{S} becomes

$$g = g_{\mu\nu} dx^\mu dx^\nu = -c^2(1-\beta^2)dt^2 + dx^2 + dy^2 + 2\omega(-ydx + xdy)dt, \quad (55)$$

which is not time-orthogonal.

In polar coordinates it is given by

$$g = -c^2(1-\beta^2)dt^2 + dr^2 + r^2 d\phi^2 + 2\omega r^2 dt d\phi. \quad (57)$$

The coordinate speed of light can be obtained from the equation $g = 0$. For the radial speed in P_\circ we get

$$\dot{r}_\circ = \pm c\sqrt{1-\beta_\circ^2}, \quad \dot{\phi}_\circ = 0, \quad (58)$$

and for the tangential velocity

$$(r\dot{\phi})_\circ = \pm c(1 \mp \beta_\circ), \quad \dot{r}_\circ = 0, \quad (59)$$

where $\beta_\circ = \omega r_\circ/c$.

We see that there is anisotropy for the tangential propagation and not for the radial one. On the other hand,

in the radial case, the value of the coordinate speed is different from c .

The synchronization parameter is given by $\epsilon = \frac{1}{2}(1 \pm \beta_\circ)$.

In order to find the coordinates of \mathcal{L}' we need the connection coefficients. The only connection coefficients different from zero are:

$$\begin{cases} \Gamma_{00}^1 = -\frac{\omega^2 x}{c^2}, & \Gamma_{02}^1 = -\frac{\omega}{c}, \\ \Gamma_{00}^2 = -\frac{\omega^2 y}{c^2}, & \Gamma_{01}^2 = +\frac{\omega}{c}. \end{cases} \quad (60)$$

The matrix $\|b_\nu^\mu\|$ of equation (1) is easily found

$$\|b_\nu^\mu\| = \Lambda R, \quad (61)$$

where

$$\|\Lambda^\mu{}_\nu\| = \begin{pmatrix} \gamma_\circ(1-\beta_\circ^2) & 0 & -\gamma_\circ\beta_\circ \\ 0 & 1 & 0 \\ 0 & 0 & \gamma_\circ \end{pmatrix}. \quad (62)$$

where $\mu, \nu = 0, 1, 2$, and

$$\|R^\mu{}_\nu\| = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(\phi_\circ) & \sin(\phi_\circ) \\ 0 & -\sin(\phi_\circ) & \cos(\phi_\circ) \end{pmatrix}, \quad (63)$$

where r_\circ , ϕ_\circ and x_\circ° , x_\circ^1 and x_\circ^2 are the coordinates of the point P_\circ .

With these matrices we have, in P_\circ

$$((b^{-1})^T g (b^{-1})) = \eta. \quad (64)$$

Substituting in equation (1) we get the transformation $\{x^\mu\} \rightarrow \{x'^\mu\}$ (see [25] for a similar transformation)

$$\begin{cases} x'^0 &= \gamma_\circ\{(1-\beta_\circ^2)\eta^\circ + \beta_\circ[\sin\phi_\circ\eta^1 - \cos\phi_\circ\eta^2] - \\ &- \beta_\circ[\cos\phi_\circ\eta^1 + \sin\phi_\circ\eta^2]\frac{\omega}{c}\eta^\circ\}, \end{cases} \quad (65)$$

$$\begin{cases} x'^1 &= \cos\phi_\circ\eta^1 + \sin\phi_\circ\eta^2 + \\ &+ \frac{\omega}{c}(\sin\phi_\circ\eta^1 - \cos\phi_\circ\eta^2)\eta^\circ - \frac{\omega^2}{2c^2}r_\circ(\eta^\circ)^2, \end{cases} \quad (66)$$

$$\begin{cases} x'^2 &= \gamma_\circ\{-\sin\phi_\circ\eta^1 + \cos\phi_\circ\eta^2 + \\ &+ \frac{\omega}{c}[\cos\phi_\circ\eta^1 + \sin\phi_\circ\eta^2]\eta^\circ\}, \end{cases} \quad (67)$$

where, as before, $\eta^\mu = x^\mu - x_\circ^\mu$, $x^\circ = ct$, $x^1 = x$, $x^2 = y$ etc.

Using the previous transformation we must remember that

$$\begin{cases} x_\circ^1 \cos\phi_\circ + x_\circ^2 \sin\phi_\circ &= r_\circ, \\ x_\circ^1 \sin\phi_\circ - x_\circ^2 \cos\phi_\circ &= 0. \end{cases} \quad (68)$$

The new metric tensor is Minkowskian, and

$$\begin{cases} g'_{\mu\nu}(x')dx'^\mu dx'^\nu = -(dx'^0)^2 + (dx'^1)^2 + (dx'^2)^2 = \\ -c^2(1 - \beta^2)dt^2 + dx^2 + dy^2 + 2\omega(-ydx + xdy)dt, \end{cases} \quad (69)$$

up to terms of order $O((x'^\mu)^2)$ in the coefficients, or even of order $O(\eta^2)$, since the relation between x' and η is homogeneous.

More precisely,

$$g'_{\mu\nu} = \eta_{\mu\nu} + O((x'^\mu)^2), \quad (70)$$

where the first order derivatives of g' are vanishing in P_o .

We may also verify that the new coordinate axis are tangent to the corresponding vectors e_α given in equations (50), (51) and (52), that is

$$e_\alpha = \frac{\partial}{\partial x'^\alpha}, \quad (71)$$

and again with these two conditions satisfied the transformation (65), (66) and (67) is unique.

The transformation of the radial and tangential components of the velocities from \mathcal{S} to \mathcal{L}' are

$$\dot{x}'_o = \frac{1}{\gamma_o} \frac{\dot{r}_o}{1 - \beta_o^2 - \frac{\beta_o}{c} r_o \dot{\phi}_o}, \quad (72)$$

$$\dot{y}'_o = \frac{r_o \dot{\phi}_o}{1 - \beta_o^2 - \frac{\beta_o}{c} r_o \dot{\phi}_o}, \quad (73)$$

where $\dot{r} = \frac{dr}{dt}$ etc., and where

$$\begin{cases} \dot{r}_o &= \dot{x}_o \cos \phi_o + \dot{y}_o \sin \phi_o, \\ r_o \dot{\phi}_o &= -\dot{x}_o \sin \phi_o + \dot{y}_o \cos \phi_o. \end{cases} \quad (74)$$

The coordinate velocities, radial and tangential, are given by equations (58) and (59). If we substitute in (72) and in (73) the values of \dot{r} and $\dot{\phi}$ given there we get for the radial proper velocity

$$\dot{x}'_o = \pm c, \quad \dot{y}'_o = 0, \quad (75)$$

and for the tangential one

$$\dot{y}'_o = \pm c, \quad \dot{x}'_o = 0, \quad (76)$$

that is once again, the anisotropy, which was present in \mathcal{S} , in \mathcal{L}' disappears.

We have verified the absence of anisotropy in the light propagation in two particular cases, for radial propagation and separately for tangential propagation. For the

general case we have only to take a linear combination of the tangential and of the radial velocities. This can be done, but, for sake of simplicity, we omit that.

Once we have made a clear distinction between the coordinates $(t; x, y)$ and the proper coordinates $(t'; x', y')$, we may understand better some puzzling aspects of the rotating disk.

We may work for instance with the set $(t; x, y)$ of coordinates and follow two closed trips at constant time t around the rim of the disk, one clockwise and the other anti-clockwise. Now, the time t is the same as that of the laboratory frame as stated by the (47), and even the distance covered is the same, since the radius is unchanged by this transformation. But the coordinate velocities, as we have shown in (59), are different, so, as has been shown in [11], the times necessary for completing a closed path are different. The Sagnac effect follows from that.

But we may work with the other set of proper variables $(t'; x', y')$, where we should understand that an infinite collection of local inertial frames is placed along a path of constant proper time. Now the speed of light is isotropic and has its value equal to c , indeed, transforming from one frame to the next, the synchronization prescription doesn't change. What now change, as shown in [6], is the length of the paths, which are now different. Again the Sagnac effect follows.

We see that no ambiguity or contradiction arises in this two pictures of the rotating disk. These are simply two different way of seeing the system, the distinction between a coordinate speed and a standard speed being peculiar to the accelerated systems, or, better, to the not time-orthogonal systems.

The distinction between coordinate and standard velocity is also stressed in [12], where various examples are discussed from the point of view of standard coordinates and of other systems of coordinates.

VII. AN APPLICATION

What we have seen until now can find a nice application to the analysis of the Selleri's paradox [29], which, to tell the truth, was already discussed by Rizzi and Tartaglia [30]. Nevertheless, we will see that the careful distinction between the two concepts of velocity allows us to have a more clear understanding of the issue.

The argument is the following: suppose that, on a rotating disk, two light rays are emitted in opposite directions along the rim of a disk, and that, after a complete tour, they come back to the starting point. We know that there will be a difference in the arrival times of the two rays. This is the well known Sagnac effect.

The disk is supposed to be in a laboratory frame, which is a inertial frame, free of gravitational effects, and equipped with Minkowski coordinates. Let us call t_o (T in our notations) the time measured in this frame and t the time measured by a clock at rest on the disk (proper time). Let the angular speed of rotation be ω and L_o be

the circumference of the disk as seen from the laboratory frame.

The relation between t_o and t is given by the usual relation

$$t_o = tF(v, \dots), \quad (77)$$

where the factor F is $\frac{1}{\sqrt{1-(v/c)^2}}$, but this is not very important, since it will cancel.

The time intervals for the light rays to complete the tour, as measured in the laboratory, are

$$L_o - x = c(t_{o2} - t_{o1}), \quad x = c(t_{o2} - t_{o1}), \quad (78)$$

where x is the arc of which is rotated the disk during the time $t_{o2} - t_{o1}$, and the first relation determines the arc covered by the ray of light, which goes in the direction contrary to the rotation. Analogous relations holds for the other ray, and, in conclusion, we can write

$$t_{o2} - t_{o1} = \frac{L_o}{c(1 + \beta)}, \quad t_{o3} - t_{o1} = \frac{L_o}{c(1 - \beta)}, \quad (79)$$

where $\beta = \omega r/c$. If we multiply by F the left-hand side of both equations we get the analogous relations for the proper times,

$$(t_2 - t_1)F = \frac{L_o}{c(1 + \beta)}, \quad (t_3 - t_1)F = \frac{L_o}{c(1 - \beta)}. \quad (80)$$

The velocities of the two rays be c_- , for the counter-rotating ray, and c_+ for the co-rotating one. For them we get

$$\frac{1}{c_-} = \frac{t_2 - t_1}{L}; \quad \frac{1}{c_+} = \frac{t_3 - t_1}{L}, \quad (81)$$

where L is the length of the rim, as seen from the disk. Taking into account equation (80), we get for the ratio of the velocities

$$\frac{c_-}{c_+} = \frac{1 + \beta}{1 - \beta}. \quad (82)$$

Observe that the factor F doesn't contribute to this ratio. This is an important point, since it means that the ratio is the same if we use inertial times t_o or proper times t .

The paradox arises in the limit $r \rightarrow \infty$, $\omega \rightarrow 0$, while keeping constant the product $v = \omega r$. In the limiting case a small portion of the disk will become equivalent to an inertial frame, and the speed of light should be c in both directions, and the ratio (82) should be 1. So there is a contradiction and so we have the paradox.

Rizzi and Tartaglia [30] observed that the speed of light on the rim is c , as they have demonstrated in [6], and so the ratio is 1 and no contradiction arises when we perform the limit $r \rightarrow \infty$.

Let us now follow the point of view developed in the previous Sections. If we follow step by step the calculation from equation (79) until equation (82), we see that, since the factor F and the length L_o cancel out, the velocities in (82) are indeed the coordinate ones, calculated with coordinate times (which by the way are the same as the inertial ones) and with the length measured in the inertial frame.

So, for coordinates velocities, the result (82) is correct, as shown in Section VI, equation (59).

At the same time, the standard velocities are always c , and the analogous ratio is 1.

On the other hand, the result (82) is not correct for the standard velocities. This can be seen if we observe that the length L , used for calculating the velocities, cannot be the same for both directions. This is because we cannot invoke any symmetry argument. These lengths have been calculated by Rizzi and Tartaglia in [6]. They are

$$L_{\pm} = L_o \sqrt{\frac{1 \pm \beta}{1 \mp \beta}}, \quad (83)$$

where L_o , as before, is the length of the rim of the disk, as measured in the laboratory, and is put in their paper equal to $2\pi r$.

If we use these values in equations (81), with L_- and L_+ replacing L in the first and in the second equation respectively, we get indeed

$$\frac{\tilde{c}_-}{\tilde{c}_+} = 1, \quad (84)$$

where \tilde{c}_- and \tilde{c}_+ are now the standard velocities for the ray propagating in the opposite direction of the rotation and in the same direction respectively.

It remains to understand what happens in the limit $r \rightarrow \infty$. But this can be easily understood if we introduce a set of coordinates on the disk.

Let us consider the transformation from the laboratory to the disk $(T, X, Y) \mapsto (t, x, y)$, where we label with (T, X, Y) the coordinates of the laboratory and with (t, x, y) those of the rotating system.

We have, as usual

$$\begin{cases} t &= T, \\ x &= X \cos(\omega T) + Y \sin(\omega T), \\ y &= -X \sin(\omega T) + Y \cos(\omega T). \end{cases} \quad (85)$$

If we consider a point P at rest on the disk, with coordinates (x, y) , as seen from the laboratory, inverting

the previous equations we can write

$$\begin{cases} T &= t, \\ X &= x \cos(\omega t) - y \sin(\omega t), \\ Y &= x \sin(\omega t) + y \cos(\omega t). \end{cases} \quad (86)$$

In terms of polar coordinates $x = r \cos \phi$, $y = r \sin \phi$ we have

$$\begin{cases} X = \frac{v}{\omega}(\cos(\omega t) \cos \phi - \sin(\omega t) \sin(\phi)), \\ Y = \frac{v}{\omega}(\sin(\omega t) \cos(\phi) + \cos(\omega t) \sin(\phi)), \end{cases} \quad (87)$$

where we have expressed the radius r in terms of ω , $r = v/\omega$, where v is the velocity at the rim. In the limit $\omega \rightarrow 0$, developing up to the first order in ω and substituting in the expression of the metric

$$g = -c^2 dT^2 + dX^2 + dY^2, \quad (88)$$

we obtain

$$g = -c^2(1 - \beta^2)dt^2 + r^2 d\phi^2 + 2v(r d\phi)dt, \quad (89)$$

where $\beta = v/c$, and the motion of the point P is along a line orthogonal to the direction determined by the angle ϕ . Without developing all the details of the motion, we may recognize the metric as that of the Galilean transformation, studied in Section IV, where $rd\phi$ takes the place of dx .

We conclude that the frame is not inertial [31], because the transformation is linear, but not Lorentzian. Indeed, a translational motion is not necessarily given by a Lorentz transformation.

As a consequence, even in the limit, the coordinate velocities need not be equal to c , and their ratio can be different from 1. Indeed it agrees with that of equation (82), as can be seen from the equations (18).

Therefore, in the limit, we get the same ratio as for finite values of r and the paradox is solved.

VIII. CONCLUSIONS

In the preceding Sections we have applied a known mathematical result concerning metric tensors to the kinematics of non-inertial systems. Given a generic system, with a given metric tensor, we may define in the neighborhood of any point of the system a coordinate set, the standard coordinates, such that the transformed metric be Minkowskian. This transformation is almost unique. In terms of these new coordinates the speed of light is certainly isotropic and has value c .

On the other hands, the original coordinates defining the system need not be Minkowskian. The speed of light measured with these coordinates is in general not isotropic and doesn't have the value c . If the system was defined starting from an inertial system, which we call laboratory system, we may see the reason for this difference. Indeed, transforming from the laboratory to the system, we necessarily induce a well definite synchronisation prescription, not necessarily the standard one (Einstein synchronization), which is only dictated by the transformation, as seen in the discussion in Section II.

We have shown all this in some examples: the Galilean transformation, the hyperbolic motion and the rotating disk. In all these cases we have exhibited the previous transformation and we have shown how, transforming from one set of coordinates to the other, the possible anisotropy of the light speed present in terms of the original coordinates, vanishes.

It is clear that the two concepts of velocity have a different physical meaning. The coordinate speed follows from the definition of a set of coordinates which, in the examples we have seen, are not globally defined. In particular, in the example of the rotating disk, the time is global, being the same as that of the laboratory, but there is a degeneration of the metric, as seen in (55): the coefficient of dt^2 vanishes when $R\omega = c$.

On the other hand, proper times and standard lengths are the times and lengths measured by standard clocks and rods, that is by an observer at rest in the non-inertial system. Their importance from an experimental and practical point of view is obvious and cannot be neglected. This means that both concepts are important in order to have a clear understanding of the argument, as stressed in [11].

IX. ACKNOWLEDGMENT

The author is indebted to L.Lusanna and P.Sona for many suggestions, for their useful criticism and for reading the manuscript

X. APPENDIX

In this Appendix we give the definition of the anisotropy parameter ϵ , quoted in Section III and in the following Sections.

In order to define the simultaneity of two events, in a given frame, we may suppose that an observer A , who is moving in an arbitrary way, has the possibility of emitting a ray of light, which will be reflected or re-emitted back to him.

Let us call the event at which this reflection happens P_B . The time t_B of this event is what we want to define.

The observer has a clock and he can measure the time of emission and the time at which the ray come back.

Let us call the first time t_A and $t_{A'}$ the second one.

The time t_B is defined by

$$t_B = t_A + \epsilon(t'_A - t_A), \quad (90)$$

where the parameter ϵ can take any value in the open interval $(0, 1)$.

The equation (90) defines the simultaneity of the event P_B with an event lying on the world-line of the observer.

The symmetric choice $\epsilon = 1/2$ corresponds to the Einstein synchronization.

It is shown, for instance in [15], that this definition of simultaneity is as correct, from the physical point of view, as the usual one, for any choice of ϵ .

This choice has an important property: together with the hypothesis of the isotropy of the one-way speed of light, it implies the isotropy of space [15].

-
- [1] M.Pauri and M.Vallisneri, "Marzke-Wheeler Coordinates for Accelerated Observers in Special Relativity", *Found.Phys.Lett.*, **13**, 401 (2000) (gr-qc/0006095).
 - [2] D.Alba and L.Lusanna, "Simultaneity, Radar 4-Coordinates and the 3+1 Point of View about Accelerated Observers in Special Relativity", gr-qc/0311058 29 Apr 2004; "Radar 4-Coordinates and Equal-Time Cauchy Surfaces for Arbitrary Accelerated Observers", Florence preprint, November 3, 2004.
 - [3] D.Bini, L.Lusanna and B.Mashhoon, "Limitations of Radar Coordinates", gr-qc/0409052, 17 Dec 2004.
 - [4] B.Mashhoon, "Limitations of spacetime measurements", *Phys.Lett.A*, **143**, (1990) 176; "Measurements Theory and General Relativity", in: *Black Holes: Theory and Observation*, edited by F.W. Hehl, C.Kiefer and R.Metzler (Springer, Berlin, 1998), pp. 269-284; see also "The hypothesis of locality and its limitations", in "Relativity in Rotating Frames", ed. G.Rizzi and M.L.Ruggiero, ed. Kluwer, 2003, pg. 43.
 - [5] R.D.Klauber, "Toward a Consistent Theory of Relativistic Rotation", in "Relativity in Rotating Frames", ed. G.Rizzo and M.L.Ruggiero, ed. Kluwer, 2004, pg.2. See also pg. 5.
 - [6] G.Rizzi and A.Tartaglia, "Speed of light on rotating platforms", gr-qc/9805089 25 May 1998, *Found.Phys.*, **28**, 1663 (1998).
 - [7] See for instance S.Weinberg, "Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity", John Wiley & Sons, 1972, pp. 72-73.
 - [8] See reference [7], pp. 85-86, Sec. 3.6.
 - [9] C.W.Misner, K.S.Thorne and J.A.Wheeler, "Gravitation", Freeman & Co., San Francisco, 1973, pg.207 and 217.
 - [10] See for instance [7], pag. 191.
 - [11] V.Petkov, "Propagation of light in non-inertial reference frames", gr-qc/9909081, 14 Dec. 2003.
 - [12] O.Wucknitz, "Sagnac effect, twin paradox and space-time topology - Time and lenght in rotating systems and closed Minkowski space-times", gr-qc/0403111 29 Mar 2004.
 - [13] R.Anderson, I.Vetharanian and G.E.Stedman, "Conventionality of Synchronisation, Gauge Dependence and Test Theories of Relativity", *Physics Reports*, **295**, 93-180, (1998).
 - [14] P.Havas, "Simultaneity, Conventionalism, General Covariance, and the Special Theory of Relativity", *General Relativity and Gravitation*, **19**, 435 (1987), and the papers quoted therein.
 - [15] M.M.Capria, "On the Conventionality of Simultaneity in Special Relativity", *Foundations of Physics*, **31**, 775 (2001).
 - [16] C.Moller, "The theory of relativity", Oxford, 1952, pg. 274.
 - [17] See reference [16], pg.250.
 - [18] See reference [16], pg. 75.
 - [19] "Relativity in Rotating Frames, Relativistic Physics in Rotating Reference Frames", edited by G.Rizzi and M.L.Ruggiero, Kluwer Academic Publisher, Dordrecht, 2004.
 - [20] A.Peres, "Synchronization of clocks in a rotating frame", *Phys.Rev.* **18**, 2173,(1978).
 - [21] V.Cantoni, "What is Wrong with Relativistic Kinematics?", *Il Nuovo Cimento*, Vol. LVII B, **1**, 220,(1968).
 - [22] O.Grøn, "Rotating Frames in Special Relativity Analyzed in Light of a Recent Article by M.Strauss", *Int.J.Theor.Phys.*, **16**, 603, (1977).
 - [23] M.Strauss, "Rotating Frames in Special Relativity", *Int.J.Theor.Phys.*, **11**, 107, (1974).
 - [24] J.Anandan, "Sagnac effect in relativistic and nonrelativistic physics", *Phys.Rev. D*, **24**, 338,(1981).
 - [25] B.Mashhoon, "The hypothesis of locality in relativistic physics", *Phys.Lett.A*, **145**, 147, (1990).
 - [26] See reference [9], pg. 166.
 - [27] See the first reference in [4].
 - [28] Ll.Bel, "Eppur si muove!", in "Relativity in Rotating Frames, Relativistic Physics in Rotating Reference Frames", edited by G.Rizzi and M.L.Ruggiero, Kluwer Academic Publisher, Dordrecht, 2004.
 - [29] F.Selleri, *Found.Phys.Lett.*, **10**, 73 (1997); "Sagnac Effect: end of a mystery", in "Relativity in Rotating Frames, Relativistic Physics in Rotating Reference Frames", edited by G.Rizzi and M.L.Ruggiero, Kluwer Academic Publisher, Dordrecht, 2004; F.Goy and F.Selleri, "Time on a Rotating Platform", *Found.Phys.Lett.*, **10**, 17 (1997).
 - [30] G.Rizzi and A.Tartaglia, "On local and global measurements of the speed of light on rotating platforms", *Found.Phys.Lett.*, **12**, 179 (1999).
 - [31] See, for instance, [19], pg. 445-447, for a discussion of this limit.